LBNL-39608
UCB-PTH-96/54
hep-th/9611149

# Modular Invariant Formulation of Multi-Gaugino and Matter Condensation*

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## Abstract

Using the linear multiplet formulation for the dilaton superfield, we construct an effective lagrangian for hidden-sector gaugino condensation in string effective field theories with arbitrary gauge groups and matter. Nonperturbative string corrections to the Kähler potential are invoked to stabilize the dilaton at a supersymmetry breaking minimum of the potential. When the cosmological constant is tuned to zero the moduli are stabilized at their self-dual points, and the *vev*'s of their F-component superpartners vanish. Numerical analyses of one- and two-condensate examples with massless chiral matter show considerable enhancement of the gauge hierarchy with respect to the $E_8$ case. The nonperturbative string effects required for dilaton stabilization may have implications for gauge coupling unification. As a comparison, we also consider a parallel approach based on the commonly used chiral formulation.

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*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797.

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## Disclaimer

# 1. Introduction

Effective Lagrangians for gaugino condensation in effective field theories from superstrings were first constructed by generalizing the work of Veneziano and Yankielowicz [1] to include the dilaton [2] and gravity [3]. These constructions used the chiral formulation for the dilaton superfield. While the resulting Lagrangian has a simple interpretation [4] in terms of the two-loop running of the gauge coupling constant, it does not respect the modular invariance [5] of the underlying superstring theory. Modular invariance was recovered [6] by adding a moduli-dependent term to the superpotential that is reminiscent of threshold corrections [7] found in some orbifold compactifications. However there is a large class of orbifolds that do not have moduli-dependent threshold corrections [8]; moreover in all orbifold models, at least part of the modular anomaly is cancelled by a Green-Schwarz counterterm [9], which must therefore be included. This has the unfortunate effect of destabilizing the dilaton.

It was recently shown how to formulate gaugino condensation using the linear multiplet [10] formulation for the dilaton superfield, both in global supersymmetry [11, 12] and in the superconformal formulation of supergravity [12]. In this case the superfield $U$ which is the interpolating field for the Yang-Mills composite superfield ($U \simeq \mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$) emerges as the chiral projection of a real vector supermultiplet $V$ whose lowest component is the dilaton field $\ell$. Using the Kähler superspace formalism of supergravity [13, 14], which we use throughout this paper, it was subsequently shown [15] how to include the Green-Schwarz term for a pure Yang-Mills $E_8$ hidden sector. In this case there are no moduli-dependent threshold corrections and there is a single constant–the $E_8$ Casimir $C$–that governs both the Green-Schwarz term and the coupling renormalization. That model was studied in detail in [16], where it was found that the dilaton can be stabilized at a phenomenologically acceptable value with broken supersymmetry if nonperturbative terms [17, 18] are included in the Kähler potential,[1] but a sufficiently large gauge hierarchy is not generated.

The advantage of the linear multiplet formulation of gaugino condensation is twofold. First, it is the correct string formulation since among the

[1] A similar observation has been made by Casas [19] in the context of the chiral formulation and without modular invariance.

massless string modes are found the dilaton and the antisymmetric tensor field. Second, the traditional chiral formulation of gaugino condensation is incorrect in that it treats the interpolating field $U \simeq \mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ as an ordinary chiral superfield of Kähler chiral weight $w = 2$. However this is inconsistent [11, 12, 15] with the constraint

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 24R^\dagger)\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) \; - \; (\mathcal{D}_{\dot\alpha} \mathcal{D}^{\dot\alpha} - 24R)\mathrm{Tr}(\mathcal{W}_{\dot\alpha} \mathcal{W}^{\dot\alpha}) \; = \; \text{total derivative}, \tag{1.1}$$

where $\mathcal{W}^\alpha$ is the Yang-Mills field strength chiral supermultiplet and the chiral superfield $R$ is an element of the super-Riemann tensor. On the other hand, the superfield $U$ considered as the chiral projection of the *real* vector superfield $V$ automatically satisfies the constraint (1.1) with $\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) \to U$. Moreover the implementation of the Green-Schwarz anomaly cancellation mechanism is simpler in the linear multiplet formulation [15] and much closer in spirit to what happens at the string level.

As mentioned above, our analysis in [16] only dealt with a pure Yang-Mills $E_8$ hidden sector. This was chosen for the purpose of illustration of the method but has several drawbacks from the point of view of phenomenology. First, the gauge coupling blows up very close to the unification scale and therefore does not allow for a large hierarchy. Second, there are no moduli-dependent threshold corrections and therefore this cannot be used to fix the vacuum expectation values of moduli fields, using for example T-duality arguments.

A more realistic situation which would involve moduli-dependent threshold corrections, would be the case of a hidden sector gauge group being a product of simple groups: $\mathcal{G} = \prod_a \mathcal{G}_a$. One immediate difficulty is the following: since we want to describe several gaugino condensates $U_a \simeq \mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)_a$, we need to introduce several vector superfields $V_a$. However, since the theory has a single dilaton $\ell$, it must be identified with the lowest component of $V = \sum_a V_a$. What should we do with the other components $\ell_a = V_a|_{\theta=\bar\theta=0}$? We will see that, in our description, these are nonpropagating degrees of freedom which actually do not appear in the Lagrangian. Similarly only one antisymmetric tensor field (also associated with $V = \sum_a V_a$) is dynamical. This allows us to generalize our approach to the case of multicondensates.

Let us stress that the goal is very different from the so-called "racetrack" ideas [20] where going to the multicondensate case is necessary in order to get supersymmetry breaking. Here supersymmetry is broken already for a single

condensate. Indeed, we will see that the picture which emerges from the multicondensate case (complete with threshold corrections and Green-Schwarz mechanism) is very different from the standard "racetrack" description: indeed, the scalar potential is largely dominated by the condensate with the largest one-loop beta-function coefficient.

To be more precise, we generalize in this paper the Lagrangian of [16] to models with arbitrary hidden sector gauge groups and with three untwisted (1,1) moduli $T^I$. We take the Kähler potential for the effective theory at the condensation scale to be:

$$K \;=\; k(V) + \sum_I g^I, \quad g^I = -\ln(T^I + \bar{T}^I), \quad V = \sum_{a=1}^{n} V_a, \tag{1.2}$$

where the $V_a$ are real vector supermultiplets and $n$ is the number of (asymptotically free) nonabelian gauge groups $\mathcal{G}_a$ in the hidden sector:

$$\mathcal{G}_{\rm hidden} = \prod_{a=1}^{n} \mathcal{G}_a \otimes U(1)^m. \tag{1.3}$$

We will take $\mathcal{G}_{\rm hidden}$ to be a subgroup of $E_8$.

We introduce both gauge condensate superfields $U_a$ and matter condensate superfields $\Pi^\alpha$ that are nonpropagating:

$$U_a \simeq \mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)_a, \quad \Pi^\alpha \simeq \prod_A \left(\Phi^A\right)^{n^A_\alpha}, \tag{1.4}$$

where $\mathcal{W}_a$ and $\Phi^A$ are the gauge and matter chiral superfields, respectively. The condensate $\Pi^\alpha$ is a chiral superfield of Kähler chiral weight $w = 0$, while the condensate $U_a$ associated with $\mathcal{G}_a$ is a chiral superfield of weight $w = 2$, and is identified with the chiral projection of $V_a$:

$$U_a \;=\; -(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)V_a, \quad \bar{U}_a \;=\; -(\mathcal{D}^\alpha\mathcal{D}_\alpha - 8R^\dagger)V_a. \tag{1.5}$$

We are thus introducing $n$ scalar fields $\ell_a = V_a|_{\theta=\bar\theta=0}$. However only one of these is physical, namely $\ell = \sum_a \ell_a$; the others do not appear in the effective component Lagrangian constructed below.

The effective Lagrangian for gaugino condensation is constructed and analyzed in Sections 2–5. In an appendix we discuss a parallel construction using the chiral supermultiplet formulation for the dilaton and unconstrained chiral supermultiplets for the gaugino condensates, in order to illustrate the differences between the two approaches. In Section 6 we summarize our results and comment on their implications for gauge coupling unification.

## 2. Construction of the effective Lagrangian

We adopt the following superfield Lagrangian:

$$\mathcal{L}_{eff} = \mathcal{L}_{KE} + \mathcal{L}_{GS} + \mathcal{L}_{th} + \mathcal{L}_{VY} + \mathcal{L}_{pot}, \tag{2.1}$$

where

$$\mathcal{L}_{KE} = \int \mathrm{d}^4\theta\, E\left[-2 + f(V)\right], \quad k(V) = \ln V + g(V), \tag{2.2}$$

is the kinetic energy term for the dilaton, chiral and gravity superfields. The functions $f(V), g(V)$ parameterize nonperturbative string effects. They are related by the condition

$$V\frac{\mathrm{d}g(V)}{\mathrm{d}V} \;=\; -V\frac{\mathrm{d}f(V)}{\mathrm{d}V} \;+\; f, \tag{2.3}$$

which ensures that the Einstein term has canonical form [16]. In the classical limit $g = f = 0$; we therefore impose the weak coupling boundary condition:

$$g(V=0) \;=\; 0 \quad \text{and} \quad f(V=0) \;=\; 0. \tag{2.4}$$

Two counter terms are introduced to cancel the modular anomaly, namely the Green-Schwarz term [9]:

$$\mathcal{L}_{GS} = b\int \mathrm{d}^4\theta\, EV\sum_I g^I, \quad b = \frac{C}{8\pi^2}, \tag{2.5}$$

and the term induced by string loop corrections [7]:

$$\mathcal{L}_{th} \;\;=\;\; -\sum_{a,I}\frac{b_a^I}{64\pi^2}\int \mathrm{d}^4\theta\,\frac{E}{R}U_a \ln\eta^2(T^I) + \text{h.c.}. \tag{2.6}$$

The parameters

$$b_a^I = C - C_a + \sum_A\left(1 - 2q_I^A\right)C_a^A, \quad C = C_{E_8}, \tag{2.7}$$

vanish for orbifold compactifications with no $N = 2$ supersymmetry sector [8]. Here $C_a$ and $C_a^A$ are quadratic Casimir operators in the adjoint and

matter representations, respectively, and $q_I^A$ are the modular weights of the matter superfields $\Phi^A$ of the underlying hidden sector theory. The term

$$\mathcal{L}_{VY} = \sum_a \frac{1}{8}\int \mathrm{d}^4\theta\, \frac{E}{R}U_a \left[b'_a \ln(e^{-K/2}U_a/\mu^3) + \sum_\alpha b_a^\alpha \ln \Pi^\alpha\right] + \text{h.c.}, \qquad (2.8)$$

where $\mu$ is a mass parameter of order one in reduced Planck units (that we will set to unity hereafter), is the generalization to supergravity [2, 3] of the Veneziano-Yankielowicz superpotential term, including [21] the gauge invariant composite matter fields $\Pi^\alpha$ introduced in Eq.(1.4) (one can also take linear combinations of such gauge invariant monomials that have the same modular weight). Finally

$$\mathcal{L}_{pot} \;=\; \frac{1}{2}\int \mathrm{d}^4\theta\, \frac{E}{R}e^{K/2}W(\Pi^\alpha, T^I) + \text{h.c.} \qquad (2.9)$$

is a superpotential for the matter condensates that respects the symmetries of the superpotential $W(\Phi^A, T^I)$ of the underlying field theory.

The coefficients $b$ in (2.8) are dictated by the chiral and conformal anomalies of the underlying field theory. Under modular transformations, we have:

$$\begin{aligned}
T^I &\rightarrow \frac{aT^I - ib}{icT^I + d}, \quad ad - bc = 1, \quad a,b,c,d \,\in \mathrm{Z}, \\
g^I &\rightarrow F^I + \bar{F}^I, \quad F^I = \ln(icT^I + d), \quad \Phi^A \rightarrow e^{-\sum_I F^I q_I^A}\Phi^A, \\
\lambda_a &\rightarrow e^{-\frac{i}{2}\sum_I \mathrm{Im}F^I}\lambda_a, \quad \chi^A \rightarrow e^{\frac{1}{2}\sum_I (i\mathrm{Im}F^I - 2q_I^A F^I)}\chi^A, \quad \theta \rightarrow e^{-\frac{i}{2}\sum_I \mathrm{Im}F^I}\theta, \\
U_a &\rightarrow e^{-i\sum_I \mathrm{Im}F^I}U_a, \quad \Pi^\alpha \rightarrow e^{-\sum_I F^I q_I^\alpha}\Pi^\alpha, \quad q_I^\alpha = \sum_A n_\alpha^A q_I^A. \qquad (2.10)
\end{aligned}$$

The field theory loop correction to the effective Yang-Mills Lagrangian from orbifold compactification has been determined [22, 23] using supersymmetric regularization procedures that ensure a supersymmetric form for the modular anomaly. Matching the variation under (2.10) of that contribution to the Yang-Mills Lagrangian with the variation of the effective Lagrangian (2.8) we require

$$\delta\mathcal{L}_{VY} = -\frac{1}{64\pi^2}\sum_{a,I}\int d^4\theta \frac{E}{R}U_a \left[C_a - \sum_{A,I} C_a^A\left(1 - 2q_I^A\right)\right] F^I + \text{h.c.}, \qquad (2.11)$$

which implies

$$b'_a + \sum_{\alpha,A} b^\alpha_a n^A_\alpha q^A_I = \frac{1}{8\pi^2}\left[C_a - \sum_A C^A_a \left(1 - 2q^A_I\right)\right] \quad \forall \ I. \tag{2.12}$$

In the flat space limit where the reduced Planck mass $m_P \to \infty$, under a canonical scale transformation

$$\lambda \to e^{\frac{3}{2}\sigma}\lambda, \quad U \to e^{3\sigma}U, \quad \Phi^A \to e^\sigma \Phi^A, \quad \Pi^\alpha \to e^{\sum_A n^A_\alpha \sigma}\Pi^\alpha, \quad \theta \to e^{-\frac{1}{2}\sigma}\theta,$$

we have the standard trace anomaly as determined by the $\beta$-functions:

$$\delta\mathcal{L}_{eff} = \frac{1}{64\pi^2}\sigma \sum_a \int d^4\theta \frac{E}{R} U_a \left(3C_a - \sum_A C^A_a\right) + \text{h.c.} + O(m_P^{-1}), \tag{2.13}$$

which requires

$$3b'_a + \sum_{\alpha,A} b^\alpha_a n^A_\alpha = \frac{1}{8\pi^2}\left(3C_a - \sum_A C^A_a\right) + O(m_P^{-1}). \tag{2.14}$$

Eqs. (2.12) and (2.14) are solved by [21] [up to $O(m_P^{-1})$ corrections]

$$b'_a = \frac{1}{8\pi^2}\left(C_a - \sum_A C^A_a\right), \quad \sum_{\alpha,A} b^\alpha_a n^A_\alpha q^A_I = \sum_A \frac{C^A_a}{4\pi^2} q^A_I, \quad \sum_{\alpha,A} b^\alpha_a n^A_\alpha = \sum_A \frac{C^A_a}{4\pi^2}. \tag{2.15}$$

Note that the above arguments do not completely fix $\mathcal{L}_{eff}$ since we can *a priori* add chiral and modular invariant terms of the form

$$\Delta\mathcal{L} = \sum_{a,\alpha} b'_{a\alpha} \int \mathrm{d}^4\theta E V_a \ln\left(e^{\sum_I q^\alpha_I g^I} \Pi^\alpha \bar{\Pi}^\alpha\right). \tag{2.16}$$

For specific choices of the $b'_{a\alpha}$ the matter condensates $\Pi^\alpha$ can be eliminated from the effective Lagrangian. However the resulting component Lagrangian has a linear dependence on the unphysical scalar fields $\ell_a - \ell_b$, and their equations of motion impose physically unacceptable constraints on the moduli supermultiplets. To ensure that $\Delta\mathcal{L}$ contains the fields $\ell_a$ only through the physical combination $\sum_a \ell_a$, we have to impose $b'_{a\alpha} = b'_\alpha$ independent of $a$. If these terms were added the last condition in (2.15) would become

$$\sum_{\alpha,A} b^\alpha_a n^A_\alpha + \sum_A b'_\alpha n^A_\alpha = \sum_A \frac{C^A_a}{4\pi^2}. \tag{2.17}$$

We shall not include such terms here.

Combining (2.7) with (2.15) gives $b_a^I = 8\pi^2 (b - b'_a - \sum_\alpha b_a^\alpha q_I^\alpha)$. Superspace partial integration gives, for $X$ any chiral superfield of zero Kähler chiral weight:

$$\frac{1}{8}\int \mathrm{d}^4\theta \,\frac{E}{R} U_a \ln X + \text{h.c.} = \int \mathrm{d}^4\theta \, E V_a \ln(X\bar{X})$$
$$-\partial_m \left( \int \mathrm{d}^4\theta \, \frac{E \ln X}{8R} \mathcal{D}_{\dot\alpha} V_a E^{\dot\alpha m} + \text{h.c.} \right), \tag{2.18}$$

where $E^{\dot\alpha m}$ is an element of the supervielbein, and the total derivative on the right hand side contains the chiral anomaly ($\propto \partial_m B^m \simeq F^a_{mn}\tilde{F}^{mn}_a$) of the F-term on the left hand side. Then combining the terms (2.2)–(2.9), the "Yang-Mills" part of the Lagrangian (2.1) can be expressed – up to a total derivatives that we drop in the subsequent analysis – as a modular invariant D-term:

$$\mathcal{L}_{eff} = \int \mathrm{d}^4\theta \, E \Bigg( -2 + f(V) + \sum_a V_a \Bigg\{ b'_a \ln(\bar{U}_a U_a / e^g V) + \sum_\alpha b_a^\alpha \ln\left(\Pi_r^\alpha \bar{\Pi}_r^\alpha\right)$$
$$- \sum_I \frac{b_a^I}{8\pi^2} \ln\left[\left(T^I + \bar{T}^I\right) |\eta^2(T^I)|^2\right] \Bigg\} \Bigg) + \mathcal{L}_{pot}, \tag{2.19}$$

where

$$\Pi_r^\alpha = \prod_A (\Phi_r^A)^{n_\alpha^A} = e^{\sum_I q_I^\alpha g^I/2} \Pi^\alpha, \quad \Phi_r^A = e^{\sum_I q_I^A g^I/2} \Phi^A, \tag{2.20}$$

is a modular invariant field composed of elementary fields that are canonically normalized in the vacuum. The interpretation of this result in terms of renormalization group running will be discussed below. We have implicitly assumed affine level-one compactification. The generalization to higher affine levels is trivial.

The construction of the component field Lagrangian obtained from (2.19) parallels that given in [16] for the case $\mathcal{G} = E_8$. Since the superfield Lagrangian is a sum of F-terms that contain only spinorial derivatives of the superfield $V_a$, and the Green-Schwarz and kinetic terms that contain $V_a$ only through the sum $V$, the unphysical scalars $\ell_a$ appear in the component Lagrangian only through the physical dilaton $\ell$. The result for the bosonic

Lagrangian is

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_B \quad = \quad & -\frac{1}{2}\mathcal{R} - (1+b\ell)\sum_I \frac{1}{(t^I+\bar{t}^I)^2}\left(\partial^m\bar{t}^I\,\partial_m t^I - \bar{F}^I F^I\right) \\
& - \frac{1}{16\ell^2}\left(\ell g_{_{(1)}}+1\right)\left[4\left(\partial^m\ell\,\partial_m\ell - B^m B_m\right) + \bar{u}u - 4e^{K/2}\ell\left(W\bar{u}+u\bar{W}\right)\right] \\
& + \frac{1}{9}\left(\ell g_{_{(1)}}-2\right)\left[\bar{M}M - b^m b_m - \frac{3}{4}\left\{\bar{M}\left(\sum_b b_b' u_b - 4We^{K/2}\right) + \text{h.c.}\right\}\right] \\
& + \frac{1}{8}\sum_a\left\{\frac{f+1}{\ell} + b_a'\ln(e^{2-K}\bar{u}_a u_a) + \sum_\alpha b_a^\alpha \ln(\pi^\alpha\bar{\pi}^\alpha)\right. \\
& \qquad \left. + \sum_I\left[bg^I - \frac{b_a^I}{4\pi^2}\ln|\eta(t^I)|^2\right]\right\}\left(F_a - u_a\bar{M} + \text{h.c.}\right) \\
& - \frac{1}{16\ell}\sum_a\left[b_a'\left(\ell g_{_{(1)}}+1\right)\bar{u}u_a - 4\ell u_a\left(\sum_\alpha b_a^\alpha\frac{F^\alpha}{\pi^\alpha} + (b_a'-b)\frac{F^I}{2\mathrm{Re}t^I}\right) + \text{h.c.}\right] \\
& + \frac{i}{2}\sum_a\left[b_a'\ln(\frac{u_a}{\bar{u}_a}) + \sum_\alpha b_a^\alpha\ln(\frac{\pi^\alpha}{\bar{\pi}^\alpha})\right]\nabla^m B_m^a \; - \; \frac{b}{2}\sum_I\frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}B_m, \\
& + \sum_{I,a}\frac{b_a^I}{16\pi^2}\left[\zeta(t^I)\left(2iB_a^m\nabla_m t^I - u_a F^I\right) + \text{h.c.}\right] \\
& + e^{K/2}\left[\sum_I F^I\left(W_I + K_I W\right) + \sum_\alpha F^\alpha W_\alpha + \text{h.c.}\right],
\end{aligned}
\tag{2.21}
$$

where

$$
\begin{aligned}
\zeta(t) \quad &= \quad \frac{1}{\eta(t)}\frac{\partial\eta(t)}{\partial t}, \quad \eta(t) = e^{-\pi t/12}\prod_{m=1}^{\infty}\left(1-e^{-2m\pi t}\right), \\
\ell \quad &= \quad V|_{\theta=\bar{\theta}=0}, \\
\sigma^m_{\alpha\dot{\alpha}}B^a_m \quad &= \quad \frac{1}{2}[\,\mathcal{D}_\alpha, \mathcal{D}_{\dot{\alpha}}\,]V_a|_{\theta=\bar{\theta}=0} \; + \; \frac{2}{3}\ell_a\sigma^m_{\alpha\dot{\alpha}}b_m, \quad B^m = \sum_a B^m_a, \\
u_a \quad &= \quad U_a|_{\theta=\bar{\theta}=0} \; = \; -(\bar{\mathcal{D}}^2 - 8R)V_a|_{\theta=\bar{\theta}=0}, \quad u = \sum_a u_a, \\
\bar{u}_a \quad &= \quad \bar{U}_a|_{\theta=\bar{\theta}=0} \; = \; -(\mathcal{D}^2 - 8R^\dagger)V_a|_{\theta=\bar{\theta}=0}, \quad \bar{u} = \sum_a \bar{u}_a, \\
-4F^a \quad &= \quad \mathcal{D}^2 U_a|_{\theta=\bar{\theta}=0}, \quad -4\bar{F}^a \; = \; \bar{\mathcal{D}}^2\bar{U}_a|_{\theta=\bar{\theta}=0}, \quad F_U = \sum_a F^a, \\
\pi^\alpha \quad &= \quad \Pi^\alpha|_{\theta=\bar{\theta}=0} \quad \bar{\pi}^\alpha \; = \; \bar{\Pi}^\alpha|_{\theta=\bar{\theta}=0}
\end{aligned}
$$

$$
\begin{aligned}
-4F^\alpha &= \mathcal{D}^2\Pi^\alpha|_{\theta=\bar\theta=0}, \quad -4\bar F^\alpha \;=\; \bar{\mathcal{D}}^2\bar\Pi^\alpha|_{\theta=\bar\theta=0},\\
t^I &= T^I|_{\theta=\bar\theta=0}, \quad -4F^I \;=\; \mathcal{D}^2T^I|_{\theta=\bar\theta=0},\\
\bar t^I &= \bar T^I|_{\theta=\bar\theta=0}, \quad -4\bar F^I \;=\; \bar{\mathcal{D}}^2\bar T^I|_{\theta=\bar\theta=0},
\end{aligned} \tag{2.22}
$$

$b_m$ and $M = (\bar M)^\dagger = -6R|_{\theta=\bar\theta=0}$ are auxiliary components of the supergravity multiplet [13]. For $n = 1,\ u_a = u,\ etc.$, (2.21) reduces to the result of [16].

The equations of motion for the auxiliary fields $b_m, M, F^I, F^a + \bar F^a$ and $F^\alpha$ give, respectively,:

$$
\begin{aligned}
b_m &= 0, \quad M \;=\; \frac{3}{4}\left(\sum_a b'_a u_a - 4We^{K/2}\right),\\
F^I &= \frac{\mathrm{Re}t^I}{2(1+b\ell)}\left\{\sum_a \bar u_a\left[(b-b'_a) + \frac{b^I_a}{2\pi^2}\zeta(\bar t^I)\mathrm{Re}t^I\right] - 4e^{K/2}\left(2\mathrm{Re}t^I\bar W_I - \bar W\right)\right\},\\
\bar u_a u_a &= \frac{\ell}{e^2}e^{g-(f+1)/b'_a\ell-\sum_I b^I_a g^I/8\pi^2 b'_a}\prod_I|\eta(t^I)|^{b^I_a/2\pi^2 b'_a}\prod_\alpha(\pi^\alpha_r\bar\pi^\alpha_r)^{-b^\alpha_a/b'_a}, \quad \pi^\alpha_r = \Pi^\alpha_r|_{\theta=\bar\theta=0},\\
0 &= \sum_a b^\alpha_a u_a + 4\pi^\alpha e^{K/2}W_\alpha \quad \forall\ \alpha.
\end{aligned} \tag{2.23}
$$

Using these, the Lagrangian (2.17) reduces to

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_B &= -\frac{1}{2}\mathcal{R} - (1+b\ell)\sum_I\frac{\partial^m\bar t^I\,\partial_m t^I}{(t^I+\bar t^I)^2} - \frac{1}{4\ell^2}\left(\ell g_{(1)}+1\right)\left(\partial^m\ell\,\partial_m\ell - B^mB_m\right)\\
&\quad - \sum_a\left(b'_a\omega_a + \sum_\alpha b^\alpha_a\phi^\alpha\right)\nabla^m B^a_m - \frac{b}{2}\sum_I\frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}B_m\\
&\quad + i\sum_{I,a}\frac{b^I_a}{8\pi^2}\left[\zeta(t^I)B^m_a\nabla_m t^I - \mathrm{h.c.}\right] - V,\\
V &= \frac{\left(\ell g_{(1)}+1\right)}{16\ell^2}\left\{\bar u u + \ell\left[\bar u\left(\sum_a b'_a u_a - 4e^{K/2}W\right) + \mathrm{h.c.}\right]\right\}\\
&\quad + \frac{1}{16(1+b\ell)}\sum_I\left|\sum_a u_a\left(b - b'_a + \frac{b^I_a}{2\pi^2}\zeta(t^I)\mathrm{Re}t^I\right) - 4e^{K/2}\left(2\mathrm{Re}t^I W_I - W\right)\right|^2\\
&\quad + \frac{1}{16}\left(\ell g_{(1)} - 2\right)\left|\sum_b b'_b u_b - 4We^{K/2}\right|^2,
\end{aligned} \tag{2.24}
$$

where we have introduced the notation

$$
u_a \;=\; \rho_a e^{i\omega_a}, \quad \pi^\alpha = \eta^\alpha e^{i\phi^\alpha}, \tag{2.25}
$$

and

$$2\phi^{\alpha} = -i\ln\left(\frac{\sum_a b_a^{\alpha} u_a \bar{W}_{\alpha}}{\sum_a b_a^{\alpha} \bar{u}_a W_{\alpha}}\right) \quad \text{if } W_{\alpha} \neq 0. \tag{2.26}$$

To go further we have to be more specific. Assume[2] that for fixed $\alpha$, $b_a^{\alpha} \neq 0$ for only one value of $a$. For example, we allow no representations $(n, m)$ with both $n$ and $m \neq 1$ under $\mathcal{G}_a \otimes \mathcal{G}_b$. Then $u_a = 0$ unless $W_{\alpha} \neq 0$ for every $\alpha$ with $b_a^{\alpha} \neq 0$. We therefore assume that $b_a^{\alpha} \neq 0$ only if $W_{\alpha} \neq 0$.

Since the $\Pi^{\alpha}$ are gauge invariant operators, we may take $W$ linear in $\Pi$:

$$W(\Pi, T) = \sum_{\alpha} W_{\alpha}(T)\Pi^{\alpha}, \quad W_{\alpha}(T) = c_{\alpha}\prod_I [\eta(T^I)]^{2(q_I^{\alpha}-1)}, \tag{2.27}$$

where $\eta(T)$ is the Dedekind function. If there are gauge singlets $M^i$ with modular weights $q_I^i$, then the constants $c_{\alpha}$ are replaced by modular invariant functions:

$$c_{\alpha} \rightarrow w_{\alpha}(M, T) = c_{\alpha}\prod_i (M^i)^{n_i^{\alpha}} \prod_I [\eta(T^I)]^{2n_i^{\alpha} q_I^i}.$$

In addition if some $M^i$ have gauge invariant couplings to vector-like representations of the gauge group

$$W(\Phi, T, M) \ni c_{iAB} M^i \Phi^A \Phi^B \prod_I [\eta(T^I)]^{2(q_I^A + q_I^B + q_I^i)},$$

one has to introduce condensates $\Pi^{AB} \simeq \Phi^A \Phi^B$ of dimension two, and corresponding terms in the effective superpotential:

$$W(\Pi, T, M) \ni c_{iAB} M^i \Pi^{AB} \prod_I [\eta(T^I)]^{2(q_I^A + q_I^B + q_I^i)}.$$

Since the $M^i$ are unconfined, they cannot be absorbed into the composite fields $\Pi$. The case with only vector-like representations has been considered in [21]. To simplify the present discussion, we ignore this type of coupling and assume that the composite operators that are invariant under the gauge symmetry (as well as possible discrete global symmetries) are at least trilinear in the nonsinglets under the confined gauge group. We further assume that there are no continuous global symmetries–such as a flavor $SU(n)_R \otimes SU(n)_L$

[2]For, *e.g.*, $\mathcal{G} = E_6 \otimes SU(3)$, we take $\Pi \simeq (27)^3$ of $E_6$ or $(3)^3$ of $SU(3)$.

whose anomaly structure has to be considered [21]. With these assumptions the equations of motion (2.23) give, using $\sum_\alpha b_a^\alpha q_I^\alpha + b_a^I/8\pi^2 = b - b_a'$,

$$\begin{aligned}
\rho_a^2 &= e^{-2b_a'/b_a} e^K e^{-(1+f)/b_a \ell - b\sum_I g^I/b_a} \prod_I |\eta(t^I)|^{4(b-b_a)/b_a} \prod_\alpha |b_a^\alpha/4c_\alpha|^{-2b_a^\alpha/b_a}, \\
\pi_r^\alpha &= -e^{-\frac{1}{2}[k+\sum_I(1-q_I^\alpha)g^I]} \frac{b_a^\alpha}{4W_\alpha} u_a, \quad b_a \equiv b_a' + \sum_\alpha b_a^\alpha. \qquad (2.28)
\end{aligned}$$

Note that promoting the second equation above to a superfield relation, and substituting the expression on the right hand side for $\Pi$ in (2.19) gives

$$\begin{aligned}
\mathcal{L}_{eff} &= \int \mathrm{d}^4\theta\, E\Bigg( -2 + f(V) + \sum_a V_a \Bigg\{ b_a \ln(\bar{U}_a U_a/e^g V) \\
&\quad - \sum_\alpha b_a^\alpha \ln\left(e^{\sum_I g^I(1-q_I^\alpha)} |4W_\alpha/b_a^\alpha|^2\right) \\
&\quad - \sum_I \frac{b_a^I}{8\pi^2} \ln\left[\left(T^I + \bar{T}^I\right) |\eta^2(T^I)|^2\right] \Bigg\}\Bigg) + \mathcal{L}_{pot}. \qquad (2.29)
\end{aligned}$$

It is instructive to compare this result with the effective Yang-Mills Lagrangian found [22, 23] by matching field theory and string loop calculations. Making the identifications $V \to L$, $U_a \to \mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)_a$, the effective Lagrangian at scale $\mu$ obtained from those results can be written:

$$\begin{aligned}
\mathcal{L}_{eff}^{YM}(\mu) &= \int \mathrm{d}^4\theta\, E\Bigg( -2 + f(V) + \sum_a V_a \Bigg\{ \frac{1}{8\pi^2}\left(C_a - \frac{1}{3}\sum_A C_a^A\right) \ln\left[\frac{\mu_s^6 g_s^{-4}}{\mu^6 g_a(\mu)^{-4}}\right] \\
&\quad - \frac{1}{4\pi^2}\sum_A C_a^A \ln\left[g_s^{\frac{2}{3}} Z_A(\mu_s)/g_a^{\frac{2}{3}}(\mu) Z_A(\mu)\right] \\
&\quad - \sum_I \frac{b_a^I}{8\pi^2} \ln\left[\left(T^I + \bar{T}^I\right) |\eta^2(T^I)|^2\right] \Bigg\}\Bigg), \qquad (2.30)
\end{aligned}$$

with $\mu_s^2 \sim g_s^2 \sim \ell$ in the string perturbative limit, $f(V) = g(V) = 0$. The first term in brackets in (2.29) can be identified with the corresponding term (2.30) provided

$$\sum_\alpha b_a^\alpha = \frac{1}{12\pi^2}\sum_A C_a^A, \quad b_a = \frac{1}{8\pi^2}\left(C_a - \frac{1}{3}\sum_A C_a^A\right). \qquad (2.31)$$

In fact, this constraint follows from (2.15) if the $\Pi^\alpha$ are all of dimension three, which is consistent with the fact that only dimension-three operators survive

in the superpotential in the limit $m_P \to \infty$. Then $b_a$ is proportional to the $\beta$-function for $\mathcal{G}_a$, and $\rho_a \simeq < \bar{\lambda}_a \lambda_a >$ has the expected exponential suppression factor for small coupling. In the absence of nonperturbative corrections to the Kähler potential $[f(V) = g(V) = 0]$, $\langle V|_{\theta=\bar{\theta}=0}\rangle = < \ell >= g_s^2 = \mu_s^2$ is the string scale in reduced Planck units and also the gauge coupling at that scale [22, 23]. Therefore the argument of the log:

$$\left\langle \left(\frac{\bar{U}_a U_a}{V}\right)^{\frac{1}{3}} \right\rangle = \frac{\left\langle \left(\bar{\lambda}_a \lambda_a\right)^{\frac{1}{3}} \right\rangle}{g_s^{\frac{2}{3}}} = \frac{\left\langle \left(\bar{\lambda}_a \lambda_a\right)^{\frac{1}{3}} \right\rangle}{\mu_s^2 g_s^{-\frac{4}{3}}} \tag{2.32}$$

gives the exact two-loop result for the coefficient of $C_a$ in the renormalization group running from the string scale to the appropriate condensate scale [4, 22, 23]. The relation between $< \pi^\alpha >$ and $< u_a >$, and hence the appearance of the gaugino condensate as the effective infra-red cut-off for massless matter loops, is related to the Konishi anomaly [24]. The matter loop contributions have additional two-loop corrections involving matter wave function renormalization [25, 26, 27]:

$$\begin{aligned} \frac{\partial \ln Z_A(\mu)}{\partial \ln \mu^2} &= -\frac{1}{32\pi^2}\Bigg[\ell e^g \sum_{BC} e^{\sum_I g^I\left(1-q_I^A-q_I^B-q_I^C\right)} Z_A^{-1}(\mu) Z_B^{-1}(\mu) Z_C^{-1}(\mu) |W_{ABC}|^2 \\ &\quad -4\sum_a g_a^2(\mu) C_2^a(R_A)\Bigg] + O(g^4) + O(\Phi^2), \end{aligned} \tag{2.33}$$

where $C_2^a(R_A) = (\mathrm{dim}\mathcal{G}_a/\mathrm{dim}R_A)C_a^A$, $R_A$ is the representation of $\mathcal{G}_a$ on $\Phi_A$. The boundary condition on $Z_A$ is [22] $Z_A(\mu_s) = (1 - p_A\ell)^{-1}$ where $p_A$ is the coefficient of $e^{\sum_I q_I^A g^I}|\Phi^A|^2$ in the Green-Schwarz counter term in the underlying field theory: $V = \sum_I g^I + p_A e^{\sum_I q_I^A g^I}|\Phi^A|^2 + O(|\Phi^A|^4)$. The second line of (2.29) can be interpreted as a rough parameterization of the second line of (2.30).

In the following analysis, we retain only dimension three operators in the superpotential, and do not include any unconfined matter superfields in the effective condensate Lagrangian. The potential takes the form

$$\begin{aligned} V &= \frac{1}{16\ell^2}\sum_{a,b}\rho_a\rho_b \cos\omega_{ab} R_{ab}(t^I), \quad \omega_{ab} = \omega_a - \omega_b, \\ R_{ab} &= \left(\ell g_{(1)} + 1\right)(1 + b_a\ell)(1 + b_b\ell) - 3\ell^2 b_a b_b + \frac{\ell^2}{(1 + b\ell)}\sum_I d_a(t^I) d_b(\bar{t}^I), \end{aligned}$$

$$
\begin{aligned}
d_a(t^I) &= b - b'_a + \frac{b^I_a}{2\pi^2}\zeta(t^I)\mathrm{Re}t^I - \sum_\alpha b^\alpha_a\left[1 - 4(q^\alpha_I - 1)\mathrm{Re}t^I\zeta(t^I)\right] \\
&= (b - b_a)\left(1 + 4\zeta(t^I)\mathrm{Re}t^I\right). \qquad (2.34)
\end{aligned}
$$

Note that $d_a(t^I) \propto F^I$ vanishes at the self-dual point $t^I = 1$, $\zeta(t^I) = -1/4$, $\eta(t^I) \approx .77$. For $\mathrm{Re}t^I \gtrsim 1$ we have, to a very good approximation, $\zeta(t^I) \approx -\pi/12$, $\eta(t^I) \approx e^{-\pi t/12}$. Note that also $\rho_a$ – and hence the potential $V$ – vanishes in the limits of large and small radii; from (2.28) we have

$$
\begin{aligned}
\lim_{t^I\to\infty}\rho^2_a &\sim (2\mathrm{Re}t^I)^{(b-b_a)/b_a}e^{-\pi(b-b_a)\mathrm{Re}t^I/3b_a}, \\
\lim_{t^I\to 0}\rho^2_a &\sim (2\mathrm{Re}t^I)^{(b_a-b)/b_a}e^{-\pi(b-b_a)/3b_a\mathrm{Re}t^I}, \qquad (2.35)
\end{aligned}
$$

where the second line follows from the first by the duality invariance of $\rho^2_a$. So there is potentially a "runaway moduli problem". However, as shown in Section 4, the moduli are stabilized at a physically acceptable vacuum, namely the self-dual point.

# 3. The axion content of the effective theory

Next we consider the axion states of the effective theory. If all $W_\alpha \neq 0$, the equations of motion for $\omega_a$ obtained from (2.24) read:

$$
\frac{\partial\mathcal{L}}{\partial\omega_a} = -b'_a\nabla^m B^a_m - \frac{1}{2}\sum_{\alpha,b} b^\alpha_b\left(\frac{b^\alpha_a u_a}{\sum_c b^\alpha_c u_c} + \mathrm{h.c.}\right)\nabla^m B^b_m - \frac{\partial V}{\partial\omega_a} = 0. \qquad (3.1)
$$

These give, in particular,

$$
\sum_a \frac{\partial\mathcal{L}}{\partial\omega_a} = -\sum_a b_a\nabla^m B^a_m = 0. \qquad (3.2)
$$

The one-forms $B^a_m$ are *a priori* dual to 3-forms:

$$
B^a_m = \frac{1}{2}\epsilon_{mnpq}\left(\frac{1}{3!4}\Gamma^{npq}_a + \partial^n b^{pq}_a\right), \qquad (3.3)
$$

where $\Gamma_a^{npq}$ and $b_a^{pq}$ are 3-form and 2-form potentials, respectively; (3.3) assures the constraints (1.1) for $\mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha) \to U_a$; explicitly

$$(\mathcal{D}^\alpha\mathcal{D}_\alpha - 24R^\dagger)U_a - (\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 24R)\bar{U}_a \;=\; -2i\,{}^*\Phi_a = -\frac{2i}{3!}\epsilon_{mnpq}\partial^m\Gamma_a^{npq} = -16i\nabla^m B_m^a. \tag{3.4}$$

We obtain

$$-\,b_a'\,{}^*\Phi_a - \frac{1}{2}\sum_{\alpha,b} b_b^\alpha\left(\frac{b_a^\alpha u_a}{\sum_c b_c^\alpha u_c} + \mathrm{h.c.}\right){}^*\Phi_b = 8\frac{\partial V}{\partial\omega_a}, \quad \sum_a b_a\,{}^*\Phi_a = 0. \tag{3.5}$$

If $\Gamma^{npq} \neq 0$, $b^{pq}$ can be removed by a gauge transformation $\Gamma^{npq} \to \Gamma^{npq} + \partial^{[n}\Lambda^{pq]}$. Thus

$$B_m^a = \frac{1}{2nb_a}\epsilon_{mnpq}\partial^n\tilde{b}^{pq} + \frac{1}{3!8}\epsilon_{mnpq}\Gamma_a^{npq}, \quad \sum_a b_a\Gamma_a^{npq} = 0, \quad \tilde{b}^{pq} = \sum_a b_a b_a^{pq}, \tag{3.6}$$

and we have the additional equations of motion:

$$\frac{\delta}{\delta\tilde{b}_{pq}}\mathcal{L}_B = 0, \quad \left(\frac{1}{b_a}\frac{\delta}{\delta\Gamma^a_{npq}} - \frac{1}{b_b}\frac{\delta}{\delta\Gamma^b_{npq}}\right)\mathcal{L}_B = 0, \quad \frac{\delta}{\delta\phi}\mathcal{L}_B \equiv \frac{\partial\mathcal{L}_B}{\partial\phi} - \nabla^m\left(\frac{\partial\mathcal{L}_B}{\partial(\nabla^m\phi)}\right), \tag{3.7}$$

which are equivalent, respectively, to

$$\epsilon_{mnpq}\sum_a\frac{1}{b_a}\nabla^n\frac{\delta}{\delta B_a^m}\mathcal{L}_B = 0, \quad \left(\frac{1}{b_a}\frac{\delta}{\delta B_a^m} - \frac{1}{b_b}\frac{\delta}{\delta B_b^m}\right)\mathcal{L}_B = 0, \tag{3.8}$$

with

$$\begin{aligned}\frac{1}{e}\frac{\delta}{\delta B_m^a}\mathcal{L}_B \;=\;& \frac{\left(\ell g_{(1)} + 1\right)}{2\ell^2}B^m + b_a'\partial^m\omega_a + \frac{1}{2}\sum_{\alpha,b} b_a^\alpha\left(\frac{b_b^\alpha u_b}{\sum_c b_c^\alpha u_c} + \mathrm{h.c.}\right)\partial^m\omega_b \\ &+ \sum_\alpha b_a^\alpha\left[\partial^m\ell\frac{\partial\phi^\alpha}{\partial\ell} + \sum_I\left(\partial^m t^I\frac{\partial\phi^\alpha}{\partial t^I} + \mathrm{h.c.}\right)\right] \\ &+ i\sum_{a,I}\frac{b_a^I}{8\pi^2}\left[\zeta(t^I)\partial^m t^I - \mathrm{h.c.}\right] - \frac{b}{2}\sum_I\frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}.\end{aligned} \tag{3.9}$$

Combining these with (3.1) and the equations of motion for $\ell$ and $t^I$, one can eliminate $B_m^a$ to obtain the equations of motion for an equivalent scalar-axion Lagrangian, with a massless axion dual to $\tilde{b}_{mn}$.

Again, these equations simplify considerably if we assume that for fixed $\alpha$, $b_a^\alpha \neq 0$ for only one value of $a$. In this case (3.1) reduces to

$$\nabla^m B_m^a = -\frac{1}{b_a}\frac{\partial V}{\partial \omega_a}, \tag{3.10}$$

and we have

$$\frac{\partial \phi^\alpha}{\partial \ell} = 0, \quad \frac{\partial \phi^\alpha}{\partial t^I} = i\zeta(t^I)\left(q_I^\alpha - 1\right), \tag{3.11}$$

if we restrict the potential to terms of dimension three with no gauge singlets $M^i$. Using $\sum_\alpha b_a^\alpha \left(q_I^\alpha - 1\right) + b_a^I/8\pi^2 = b - b_a$ gives:

$$\begin{aligned}\frac{1}{e}\frac{\delta}{\delta B_m^a}\mathcal{L}_B &= \frac{\left(\ell g_{(1)} + 1\right)}{2\ell^2}B^m + b_a\partial^m\omega_a + i\sum_I\left\{\partial^m t^I\left[\zeta(t^I)\left(b - b_a\right) + \frac{b}{4\mathrm{Re}t^I}\right] - \mathrm{h.c.}\right\} \\ &\approx \frac{\left(\ell g_{(1)} + 1\right)}{2\ell^2}B^m + b_a\partial^m\omega_a + \sum_I\partial^m\mathrm{Im}t^I\left[\left(b - b_a\right)\frac{\pi}{6} - \frac{b}{2\mathrm{Re}t^I}\right],\end{aligned} \tag{3.12}$$

where the last line corresponds to the approximation $\zeta(t^I) \approx -\pi/12$. In the following we illustrate these equations using specific cases.

## A. Single condensate

As in [16] there is an axion $\omega = \omega_a + (\pi/6)(b/b_a - 1)\sum_I \mathrm{Im}t^I$ that has no potential, and, setting

$$B_a^m = \frac{1}{2}\epsilon^{mnpq}\partial_n b_{pq} = -\frac{2\ell^2}{\left(\ell g_{(1)} + 1\right)}\left(b_a\partial^m\omega - \frac{b}{2}\sum_I\frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}\right), \tag{3.13}$$

the equations of motion derived from (2.24) are equivalent to those of the effective scalar Lagrangian:

$$\begin{aligned}\frac{1}{e}\mathcal{L}_B &= -\frac{1}{2}\mathcal{R} - (1 + b\ell)\sum_I\frac{\partial^m\bar{t}^I\,\partial_m t^I}{(t^I + \bar{t}^I)^2} - \frac{1}{4\ell^2}\left(\ell g_{(1)} + 1\right)\partial^m\ell\,\partial_m\ell - V(\ell, t^I, \bar{t}^I) \\ &- \frac{\ell^2}{\left(\ell g_{(1)} + 1\right)}\left(b_a\partial^m\omega - \frac{b}{2}\sum_I\frac{\partial^m\mathrm{Im}t^I}{\mathrm{Re}t^I}\right)\left(b_a\partial_m\omega - \frac{b}{2}\sum_I\frac{\partial_m\mathrm{Im}t^I}{\mathrm{Re}t^I}\right).\end{aligned} \tag{3.14}$$

## B. Two condensates: $b_1 \neq b_2$

Making the approximation $\eta(t) \approx e^{-\pi t/12}$, the Lagrangian (2.24) can be written

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_B &= -\frac{1}{2}\mathcal{R} - (1+b\ell)\sum_I \frac{\partial^m \bar{t}^I\,\partial_m t^I}{(t^I+\bar{t}^I)^2} - \frac{1}{4\ell^2}\left(\ell g_{(1)}+1\right)\left(\partial^m\ell\,\partial_m\ell - B^m B_m\right) \\
&\quad - \omega\nabla^m \tilde{B}_m - \omega'\nabla^m B_m - \frac{b}{2}\sum_I \frac{\partial^m \mathrm{Im}t^I}{\mathrm{Re}t^I}B_m - V,
\end{aligned} \tag{3.15}
$$

where

$$
\begin{aligned}
\omega &= \frac{b_1\omega_1 - b_2\omega_2}{b_1-b_2} - \frac{\pi}{6}\sum_I \mathrm{Im}t^I, \quad \omega' = -\frac{\omega_{12}}{\beta} + \frac{b\pi}{6}\sum_I \mathrm{Im}t^I, \\
\beta &= \frac{b_1-b_2}{b_1 b_2}, \quad \tilde{B}^m = \sum_a b_a B_a^m.
\end{aligned} \tag{3.16}
$$

We have

$$
\begin{aligned}
\omega_1 &= \omega + \frac{\pi}{6}\sum_I \mathrm{Im}t^I + \frac{1}{b_1}\left(\omega' - \frac{b\pi}{6}\sum_I \mathrm{Im}t^I\right), \\
\omega_2 &= \omega + \frac{\pi}{6}\sum_I \mathrm{Im}t^I + \frac{1}{b_2}\left(\omega' - \frac{b\pi}{6}\sum_I \mathrm{Im}t^I\right), \\
\frac{\partial V}{\partial\omega_1} &= -\frac{\partial V}{\partial\omega_2} = \frac{\partial V}{\partial\omega_{12}}.
\end{aligned} \tag{3.17}
$$

Then taking $\omega, \omega'$ and $t^I$ as independent variables, the equations of motion for $\omega, \omega'$ are

$$
\begin{aligned}
\nabla^m \tilde{B}_m &= 0, \quad \tilde{B}_m = \frac{1}{2}\epsilon_{mnpq}\partial^n \tilde{b}^{pq}, \\
\nabla^m B_m &= \frac{1}{8}{}^*\Phi = \beta\frac{\partial V}{\partial\omega_{12}}, \quad B_m = \frac{1}{3!8}\epsilon_{mnpq}\Gamma^{npq}.
\end{aligned} \tag{3.18}
$$

Substituting the first of these into the Lagrangian (3.15), we see that the axion $\omega$ and the three-form $\tilde{B}_m$ drop out because they appear only linearly in the Lagrangian; hence they play the role of Lagrange multipliers. The equation of motion for $\tilde{b}_{mn}$ implies the constraint on the phase $\omega$:

$$
\nabla_m \partial^m \omega = 0. \tag{3.19}
$$

The equations of motion for $\mathrm{Im}t^I$ and $\Gamma_{mnp}$ read:

$$\begin{aligned} 0 &= \nabla_m \left[ (1+b\ell) \frac{\partial^m \mathrm{Im}t^I}{2\left(\mathrm{Re}t^I\right)^2} + \frac{b}{2\mathrm{Re}t^I} B^m \right] - i \left( \frac{\partial V}{\partial t^I} - \mathrm{h.c.} \right) - \frac{b\pi}{48} {}^*\Phi, \\ 0 &= \frac{\left(\ell g_{(1)} + 1\right)}{2\ell^2} B^m + \partial^m \omega' - \frac{b}{2} \sum_I \frac{\partial^m \mathrm{Im}t^I}{\mathrm{Re}t^I}, \end{aligned} \tag{3.20}$$

and the equivalent scalar Lagrangian is

$$\begin{aligned} \frac{1}{e}\mathcal{L}_B &= -\frac{1}{2}\mathcal{R} - (1+b\ell) \sum_I \frac{\partial^m \bar{t}^I \, \partial_m t^I}{(t^I + \bar{t}^I)^2} - \frac{1}{4\ell^2} \left(\ell g_{(1)} + 1\right) \partial^m \ell \, \partial_m \ell \\ & - \frac{\ell^2}{\left(\ell g_{(1)} + 1\right)} \left( \partial^m \omega' - \frac{b}{2} \sum_I \frac{\partial^m \mathrm{Im}t^I}{\mathrm{Re}t^I} \right) \left( \partial_m \omega' - \frac{b}{2} \sum_I \frac{\partial_m \mathrm{Im}t^I}{\mathrm{Re}t^I} \right) \\ & - V(\ell, t^I, \bar{t}^I, \omega_{12}). \end{aligned} \tag{3.21}$$

As in Subsection A, there is a single dynamical axion $\omega'$ – or, via a duality transformation, ${}^*\Phi$ – but there is now a potential for the axion.

## C. General case

We introduce $n$ linearly independent vectors $\tilde{B}_m, B_m, \hat{B}^i_m,\ i = 1 \ldots n-2$, and decompose the $B^m_a$ as

$$B^m_a = a_a \tilde{B}^m + c_a B^m + \sum_i d^i_a \hat{B}^m_i, \quad \hat{B}^m_i = \sum_a e^a_i B^m_a. \tag{3.22}$$

Then

$$\sum_a \left[ b_a \omega_a + (b - b_a) \frac{\pi}{6} \sum_I \mathrm{Im}t^I \right] \nabla_m B^m_a = \omega \nabla_m \tilde{B}^m + \omega' \nabla_m B^m + \sum_i \omega^i \nabla_m \hat{B}^m_i,$$

$$\omega_a = \omega + \frac{\pi}{6} \sum_I \mathrm{Im}t^I + \frac{1}{b_a} \left( \omega' - \frac{b\pi}{6} \sum_I \mathrm{Im}t^I \right) + \sum_i \frac{e^a_i}{b_a} \omega^i, \tag{3.23}$$

and the Lagrangian can be written as in (3.15) with an additional term:

$$\frac{1}{e}\mathcal{L}_B \to \frac{1}{e}\mathcal{L}_B - \sum_i \omega^i \nabla_m \hat{B}^m_i, \tag{3.24}$$

The equations of motion for the phases $\omega$ are:

$$\begin{aligned}
\nabla_m \tilde{B}^m &= -\frac{\partial V}{\partial \omega} = -\sum_a \frac{\partial V}{\partial \omega_a} = 0, \\
\nabla_m B^m &= -\frac{\partial V}{\partial \omega'} = -\sum_a \frac{1}{b_a}\frac{\partial V}{\partial \omega_a} = \frac{1}{2}\sum_{ab} \beta_{ab}\frac{\partial V}{\partial \omega_{ab}} = \frac{1}{8}{}^*\Phi, \quad \beta_{ab} \equiv \frac{b_a - b_b}{b_a b_b} \\
\nabla_m \hat{B}^m_i &= -\frac{\partial V}{\partial \omega^i} = -\sum_a \frac{e^a_i}{b_a}\frac{\partial V}{\partial \omega_a} = \frac{1}{8}{}^*\Phi_i,
\end{aligned} \tag{3.25}$$

and the equations for $\Gamma^i_{mnp} = 8\epsilon_{mnpq}\hat{B}^q_i$ give $\partial^m \omega^i = 0$. Hence

$$\omega_{ab} = -\beta_{ab}\left(\omega' - \frac{b\pi}{6}\sum_I \mathrm{Im} t^I\right) + \theta_{ab}, \quad \theta_{ab} = \text{constant}. \tag{3.26}$$

Thus as in the two-condensate case of Subsection B, there is one dynamical axion with a potential. The dual scalar Lagrangian is the same as (3.21), with $V = V(\ell, t^I, \bar{t}^I, \omega_{ab})$.

# 4. The effective potential

The potential (2.34) can be written in the form

$$\begin{aligned}
V &= \frac{1}{16\ell^2}\left(v_1 - v_2 + v_3\right), \\
v_1 &= \left(1 + \ell g_{(1)}\right)\left|\sum_a \left(1 + b_a \ell\right) u_a\right|^2, \quad v_2 = 3\ell^2\left|\sum_a b_a u_a\right|^2, \\
v_3 &= \frac{\ell^2}{(1 + b\ell)}\sum_I \left|\sum_a d_a(t^I) u_a\right|^2 = 4\ell^2\left(1 + b\ell\right)\sum_I \left|\frac{F^I}{\mathrm{Re} t^I}\right|^2.
\end{aligned} \tag{4.1}$$

In the strong coupling limit

$$\lim_{\ell\to\infty} V = \left(\ell g_{(1)} - 2\right)\left|\sum_a b_a u_a\right|^2, \tag{4.2}$$

giving the same condition on the functions $f, g$ as in [16] to assure boundedness of the potential. Note however that if $v_1 = v_3 = 0$ has a solution

with $v_2 \neq 0$, the vacuum energy is always negative. $v_3 = 0$ is solved by $t^I = 1$, *i.e.* the self-dual point. As explained below this is the only nontrivial minimum if the cosmological constant is fine-tuned to vanish. In the case of two condensates, there is no solution to $v_1 = 0,\ v_2 \neq 0$, for $f \geq 0$, and the cosmological constant can be fine-tuned to vanish, as will be illustrated below in a toy example. More generally, the potential is dominated by the condensate with the largest one-loop $\beta$-function coefficient, so the general case is qualitatively very similar to the single condensate case, and it appears that positivity of the potential can always be imposed. Otherwise, one would have to appeal to another source of supersymmetry breaking to cancel the cosmological constant, such as a fundamental 3-form potential [28] whose field strength is dual to the constant that has been previously introduced in the superpotential [29], and/or an anomalous $U(1)$ gauge symmetry [30].

In the following we study $Z_3$-inspired toy models with $E_6$ and/or $SU(3)$ gauge groups in the hidden sector, and $3N_f$ matter superfields [31] in the fundamental representation $f$. Asymptotic freedom requires $N_{27} \leq 3$ and $N_3 \leq 5$. For a true $Z_3$ orbifold there are no moduli-dependent threshold corrections: $b_a^I = 0$. In this case universal anomaly cancellation determines the average value of the matter modular weights in these toy models as: $< 2q_I^{27} - 1 >= 2/N_{27},\ < 2q_I^3 - 1 >= 18/N_3$. In some models Wilson line breaking of the hidden sector $E_8$ generates vector-like representations that could acquire masses above the condensation scale, so that the universal anomaly cancellation sum rule is not saturated by light states alone. In this case the $q_I^\alpha$ no longer drop out of the equations, so some of the above formulae would be slightly modified. In addition, one would have to include threshold effects [23, 32], unless the masses of the heavy states are pushed to the string scale. Here we assume for simplicity that the sum rule is saturated by the light states. Denoting the fundamental matter fields by $\Phi_f^{I\alpha},\ \alpha = 1, \ldots, N_f$, the matter condensates can be constructed as

$$\Pi_f^\alpha = \prod_{I=1}^{3} \Phi_f^{I\alpha}, \quad b_{E_6}^\alpha = \frac{3}{4\pi^2}, \quad b_{SU(3)}^\alpha = \frac{1}{8\pi^2},$$

where gauge indices have been suppressed.

In the analysis of the models described below, we assume–for obvious phenomenological reasons–that the vacuum energy vanishes at the minimum

$< V >= 0$. Thus we solve the equations

$$V = \frac{\partial V}{\partial x} = 0, \quad x = \ell, t^I, \omega_a. \tag{4.3}$$

For $x = \ell, t^I$, we have

$$\begin{aligned}
\frac{\partial \rho_a}{\partial x} &= \frac{1}{2}\left(A_x + \frac{1}{b_a}B_x\right)\rho_a, \quad B_\ell = \frac{(1+\ell g_{(1)})}{\ell^2}, \quad B_I = \frac{b}{2\mathrm{Re}t^I}\left[1 + 4\zeta(t^I)\mathrm{Re}t^I\right], \\
\frac{\partial V}{\partial x} &= \left(A_x - \frac{2}{\ell}\delta_{x\ell}\right)V + \frac{1}{16\ell^2}\sum_{ab}\rho_a\rho_b\cos\omega_{ab}\left(\frac{B_x}{b_a}R_{ab} + \frac{\partial}{\partial x}R_{ab}\right) \\
&= \frac{1}{16\ell^2}\sum_{ab}\rho_a\rho_b\cos\omega_{ab}\left(\frac{B_x}{n}\sum_c\beta_{ca}R_{ab} + \frac{\partial}{\partial x}R_{ab}\right) \\
&\qquad + \left(A_x - \frac{2}{\ell}\delta_{x\ell} + \frac{B_x}{n}\sum_a\frac{1}{b_a}\right)V,
\end{aligned} \tag{4.4}$$

where $\beta_{ab}$ is defined in (3.25). By assumption, the last term in (4.4) vanishes in the vacuum. Note that the self-dual point, $d_a(t^I) = B_I = 0,\ t^I = 1$, is always a solution to the minimization equations for $t^I$. It is the only solution for the single condensate case. For the multicondensate case, if we restrict our analysis to the (relatively) weak coupling region, $\ell < 1/b_-$, where $b_-$ is the smallest $\beta$-function constant, the potential is dominated by the condensate with the largest $\beta$-function coefficient $b_+ :\ V \approx \rho_+^2 R_{++}/16\ell^2$. Moreover, since $\pi b/3b_a > 1$, the potential is always dominated by this term for $\mathrm{Re}t^I > 1$ [*c.f.* Eq. (2.35)], so the only minimum for $\mathrm{Re}t^I > 1$ is $\mathrm{Re}t^I \to \infty,\ \rho_a \to 0$. By duality the only minimum for $\mathrm{Re}t^I < 1$ is $\mathrm{Re}t^I \to 0,\ \rho_a \to 0$, so the self-dual point is the only nontrivial solution. Since our potential is always dominated by one condensate, the picture is very different from the "race-track" models studied previously [20].

At a self-dual point with $V = 0$, we have

$$\begin{aligned}
\frac{\partial^2 V}{\partial (t^I)^2} &\approx \frac{1}{32\ell^2}\sum_{ab}\rho_a\rho_b\cos\omega_{ab}\left(\frac{\pi^2}{9}\frac{\ell^2}{(1+b\ell)}(b-b_a)(b-b_b) - \frac{b\pi}{6n}\sum_c\beta_{ca}R_{ab}\right) \\
&\approx \frac{\rho_+^2}{32}\left(\frac{\pi^2}{9}\frac{(b-b_+)^2}{(1+b\ell)} - \frac{b\pi}{6n\ell^2}\sum_c\beta_{c+}R_{++}\right).
\end{aligned} \tag{4.5}$$

Positivity of the potential requires $R_{++} \geq 0$, and $\beta_{c+} \leq 0$ by definition, so the extremum at a self-dual point with $V = 0,\ \rho_+ \neq 0$ is a true minimum.

In practice, the last term is negligible, and the normalized moduli squared mass is

$$m_{t^I}^2 \approx \left\langle \frac{\pi^2 \rho_+^2}{36} \frac{(b-b_+)^2}{(1+b\ell)^2} \right\rangle . \tag{4.6}$$

## A. Single condensate with matter

In this case $\beta_{ab}=0$, and the minimization equations for $t^I$ require

$$\frac{\partial}{\partial t^I}\left|1+4\zeta(t^I)\mathrm{Re}t^I\right|^2 = 0,$$

which is solved by $1+4\zeta(t^I)\mathrm{Re}t^I = 0,\ t^I=1$. Then $v_3 = F^I = 0$, and the potential is qualitatively the same as in the $E_8$ case [16]–except for the fact that the moduli are fixed. (Note however that if $\beta_{ab}=0$ one can choose the $b'_{a\alpha}$ in (2.16) such that the matter composites drop out of the effective Lagrangian; then $R_{aa}$ is independent of the moduli which remain undetermined.) The quantitative difference from the $E_8$ case is the value of the $\beta$-function coefficient: $b_{E_6} = (12-3N_{27})/8\pi^2,\ b_{SU(3)} = (6-N_3)/16\pi^2$. As in [16] we take the nonperturbative contribution to the dilaton Kähler potential to be of the form [17] $f = Ae^{-B/\ell}$ or [18] $f = Ae^{-B/\sqrt{\ell}}$, and fine tune the constant $A$ to get a vanishing cosmological constant.

Attention has been drawn to the leading correction for small coupling that is of the form $f = Ae^{-B/\sqrt{V}}$. If we restrict $f$ to this form we have to require a rather large value for the coefficient: $A \simeq 40$ to cancel the cosmological constant. On the other hand the important feature of $f$ here is its behaviour in the strong coupling limit; if $f$ contains terms of the form $Ae^{-B/V^{\frac{n}{2}}}$ the strong coupling limit will be dominated by the term with the largest value of $n$. In the numerical analysis we take $f = Ae^{-B/V}$; adding to this a term of the form $f = A'e^{-B'/\sqrt{V}}$ will not significantly affect the analysis. We find that the *vev* of $\ell$ is insensitive to the content of the hidden sector; it is completely determined by string nonperturbative effects, provided a potential for $\ell$ is generated by the strongly coupled hidden Yang-Mills sector. More specifically, taking $f = Ae^{-B/V}$ we find that $<V>=0$ requires $A \approx e^2 \approx 7.4$, and the dilaton is stabilized at a value $<\ell> \approx B/2$. Taking $B=1$ gives $<\ell> \approx 0.5,\ <f(\ell)> \approx 1$, and the squared gauge coupling at the string scale is $g_s^2 = <2\ell/(1+f)> \approx 0.5$. If instead we use $f = Ae^{-B/\sqrt{V}}$,

the corresponding numbers are $A \approx 2e^3 \approx 40,\ < \ell > \approx B^2/9,\ g_s^2 \approx 2B^2/27$. From now on we take $f = Ae^{-1/V}$.

The potential for $\mathcal{G}_a = E_6,\ N_{27} = 1$, is plotted in Figures 1–3. Fig. 1 shows the potential in the $\ell, \ln t$ plane, where we have set $t^I = t$, $\mathrm{Im}t = 0$; with this choice of variables the $t$-duality invariance of the potential is manifest. Fig. 2 shows the potential for $\ell$ at the self-dual point $t^I = 1$, and Fig. 3 shows the potential for $\ln t$ with $\ell$ fixed at its *vev*. The qualitative features of the potential are independent of the content of the hidden sector. Fixing $A$ in each case by the condition $< V >= 0$, we find for $\mathcal{G}_a = E_6$

$$A = \begin{cases} 7.324 \\ 7.359 \\ 7.381 \end{cases}, \quad < \ell >= \begin{cases} 0.502 \\ 0.501 \\ 0.500 \end{cases} \approx g_s^2, \quad \text{for} \quad N_{27} = \begin{cases} 1 \\ 2 \\ 3 \end{cases}. \tag{4.7}$$

For $\mathcal{G}_a = SU(3),\ N_3 = 1$, we find $A = 7.383,\ < \ell >= 0.500 \approx g_s^2$. As discussed in Section 5, the scale of supersymmetry breaking in this case is far too small, and decreases with increasing $N_3$.

## B. Two condensates

We have

$$\begin{aligned} \frac{\partial V}{\partial \omega_1} &= -\frac{\partial V}{\partial \omega_2} = -\rho_1\rho_2 R_{12} \sin\omega_{12}, \\ \sum_{abc} \beta_{ca}\rho_a\rho_b R_{ab} \cos\omega_{ab} &= \beta_{21}\left(\rho_1^2 R_{11} - \rho_2^2 R_{22}\right). \end{aligned} \tag{4.8}$$

Minimization with respect to $\omega_1$ requires either $< \sin\omega_{12} >= 0$ or $< R_{12} >= 0$. Identifying $b_1 = b_+,\ b_2 = b_-$, positivity of the potential requires $R_{11} \geq 0$, which in turn implies $R_{12} > 0$, so the extrema in $\omega$ are at $\sin\omega_{12} = 0$, with $\cos\omega_{12} = -1(+1)$ corresponding to minima (maxima):

$$\frac{\partial^2 V}{\partial \omega_{12}^2} = -\rho_1\rho_2 R_{12}\cos\omega_{12}, \quad m^2_{\omega_{12}} = \left\langle \frac{3b_+^2\beta^2}{2(1+b_+\ell)^2}\rho_1\rho_2 R_{12} \right\rangle. \tag{4.9}$$

There is also a small $\mathrm{Im}t^I$-$\omega_{12}$ mixing. Note that while in contrast to the single condensate case, the dynamical axion is no longer massless, its mass is exponentially suppressed relative to the gravitino mass by a factor $\sim \sqrt{\rho_2/\rho_1}$.

We do not expect this feature to persist when kinetic terms are introduced for the condensate fields.

For $\mathcal{G} = E_6 \otimes SU(3)$, the potential is dominated by the $E_6$ condensate, and the results are the same as in (4.7). The only other gauge groups in the restricted set considered here that are subgroups of $E_8$ are $\mathcal{G} = [SU(3)]^n,\ n \leq 4$; these cannot generate sufficient supersymmetry breaking.

# 5. Supersymmetry breaking

The pattern and scale of supersymmetry breaking are determined by the *vev*'s of the $F$-components of the chiral superfields. From the equations of motion for $\pi^\alpha$ and $\rho_a$ we obtain, at the self-dual point $< F^I >= 0$:

$$\begin{aligned}
\langle F^\alpha \rangle &= \frac{(\ell g_{_{(1)}} + 1)}{4\ell^2 b_a}\pi^\alpha \left(\bar{u} + \ell \sum_b b_b \bar{u}_b\right) \approx \frac{3b_+^2}{4b_a}\pi^\alpha \bar{u}_+ \left(1 + \ell b_+\right)^{-1}, \quad b_a^\alpha \neq 0, \\
\langle F^a + \bar{F}^a \rangle &= \frac{1}{4\ell^2 b_a}(\ell g_{_{(1)}} + 1)(1 + \ell b_a)\left[u_a \left(\bar{u} + \ell \sum_b b_b \bar{u}_b\right) + \text{h.c.}\right] \\
&\approx \frac{3b_+^2}{4b_a}\frac{1 + \ell b_a}{1 + \ell b_+}\left(u_a \bar{u}_+ + \bar{u}_a u_+\right),
\end{aligned} \tag{5.1}$$

where the approximations on the right hand sides are exact for a single condensate. The dominant contribution is from the condensate with the largest $\beta$-function coefficient:

$$\langle F^+ + \bar{F}^+ \rangle = \frac{3\rho_+^2 b_+}{2}. \tag{5.2}$$

The fact that the $F^I$ vanish in the vacuum is a desirable feature for phenomenology. Nonuniversal squark and slepton masses that could induce unacceptably large flavor-changing neutral currents are thereby avoided. However this feature might be modified in the presence of moduli-dependent threshold effects $\sim \ln(\mu^2)$ where $\mu^2 =< e^{\sum_I q_I^i g^I} |M^i|^2 >$ is a modular invariant squared mass and $M^i$ is a gauge singlet with modular weights $q_i^I$.

Another important parameter for soft supersymmetry breaking in the observable sector is the gravitino mass. The derivation of the gravitino part of the Lagrangian again parallels the construction in [16]. The gravitino mass

$m_{\tilde{G}}$ is determined by the term

$$\begin{aligned}\mathcal{L}_{mass}(\psi) &= -\frac{1}{8}\psi^m\sigma_{mn}\psi^n\sum_a \bar{u}_a\left\{\frac{f+1}{\ell} + b'_a\ln(e^{2-K}\bar{u}_a u_a) + \sum_\alpha b_a^\alpha \ln(\pi^\alpha\bar{\pi}^\alpha)\right.\\ &\left. + \sum_I\left[bg^I - \frac{b_a^I}{4\pi^2}\ln|\eta(t^I)|^2\right]\right\} - e^{K/2}\bar{W}\psi^m\sigma_{mn}\psi^n + \text{h.c.}, \quad (5.3)\end{aligned}$$

giving, when the equations of motion (2.23) are imposed,

$$m_{\tilde{G}} = \frac{1}{3}\langle|M|\rangle = \frac{1}{4}\left\langle|\sum_a b'_a u_a - 4e^{K/2}W|\right\rangle = \frac{1}{4}\left\langle|\sum_a b_a u_a|\right\rangle \approx \frac{1}{4}b_+\langle\rho_+\rangle. \quad (5.4)$$

The scale of supersymmetry breaking is governed by the *vev* (2.28) of the condensate with the largest $\beta$-function coefficient. This includes the usual suppression factor $<\rho_a>\propto e^{-1/b_a g_s^2}$, where $g_s^2 =< 2\ell/(1+f) >$ is the effective squared coupling constant at the string scale. However there are other important parameters that determine the scale of the hierarchy between the supersymmetry breaking scale and the Planck scale. The dependence on the moduli provides a second exponential suppression factor:

$$<\rho_a>\propto<\prod_I|\eta(t^I)|^{2(b-b_a)/b_a}>=|\eta(1)|^{6(b-b_a)/b_a}\approx e^{-\pi(b-b_a)/2b_a}. \quad (5.5)$$

On the other hand, the numerical factor $\prod_\alpha|b_a^\alpha/4c_\alpha|^{-b_a^\alpha/b_a}$ gives an exponential enhancement if $c_\alpha\sim 1$. This is the largest numerical uncertainty in our analysis. *A priori*, $c_\alpha$ is related to the Yukawa couplings for matter in the hidden sector. However, there is an arbitrary normalization factor in the definition of $\Pi^\alpha$. If the hidden sector Yukawa couplings were known, it might be possible to estimate $c_\alpha$ by a matching condition for the *vev*'s of the second lines of (2.29) and (2.30). In our numerical analysis we have set $c_\alpha = 1$. Then if the hidden gauge group with the largest condensate is $\mathcal{G}_+ = E_6$ with $3N_{27}$ matter chiral superfields in the fundamental representation, we obtain

$$m_{\tilde{G}} = \begin{cases}1.1\times10^{-9}\\ 3.3\times10^{-11}\\ 1.65\times10^{-15}\end{cases} \quad \text{for} \quad N_{27} = \begin{cases}1\\2\\3\end{cases}, \quad (5.6)$$

in reduced Planck units. For $\mathcal{G}_+ = SU(3)$ with three matter chiral fields in the fundamental representation, we obtain an unacceptably large gauge hierarchy: $m_{\tilde{G}} = 2.2\times10^{-32}$; $m_{\tilde{G}}$ decreases rapidly as $N_3$ increases, *i.e.* as the $\beta$-function coefficient decreases.

# 6. Concluding remarks

In the class of models studied here, the introduction of a parameterization for nonperturbative contributions to the Kähler potential for the dilaton generically allows a stable vacuum at a nontrivial, phenomenologically acceptable point in the dilaton/moduli space. In particular, when we impose the constraint that the cosmological constant vanishes, we find that in the linear multiplet formulation, the moduli $t^I$ are stabilized at the self-dual point, and their associated auxiliary fields vanish in the vacuum, which implies the phenomenologically desirable feature of universal soft supersymmetry breaking parameters. As shown in the Appendix, these features do not survive in the parallel construction starting from the chiral multiplet formalism because of the explicit $s$-dependence of the superpotential. They may also be modified in the linear multiplet formalism in the presence of moduli-dependent intermediate-scale threshold effects. However the case with no such threshold corrections serves to illustrate the difference between the two approaches. We have argued that the linear multiplet approach more faithfully respects the physics of the underlying strongly coupled Yang-Mills theory.

A salient feature of our formalism is that there is little qualitative difference between a single condensate and a multi-condensate scenario. For several condensates with equal (or very similar) $\beta$-functions, the potential reduces to that of the single condensate case, except that there may be flat directions. If $b_1 = b_2 = \cdots b_k$, then at the self-dual point $\rho_a/\rho_1 = \zeta_a =$ constant and the potential vanishes identically in the direction $\sum_{a=1}^{k} \zeta_a e^{i\omega_a} = 0,\ \rho_{a>k} = 0$. This always has a solution if $\zeta_a = 1$, in which case the flat direction preserves supersymmetry and there is no barrier between this solution and the interesting, supersymmetry breaking solution. For different $\beta$-functions, the potential is dominated by the condensate(s) with the largest $\beta$-function coefficient, and the result is essentially the same as in the single condensate case, except that a small mass is generated for the dynamical axion. In all cases nonperturbative corrections to the dilaton Kähler potential are required to stabilize the dilaton. This picture is very different from previously studied "racetrack" models [20] where dilaton stabilization is achieved through cancellations among different condensates with similar $\beta$-functions. The qualitative difference between an $E_8$ hidden sector and one with a product gauge group is the presence of matter; in the $E_8$ case the potential is independent of the moduli, which therefore remain undetermined

in the classical vacuum of the effective condensate theory.

As discussed previously [11, 12, 16], kinetic energy terms for the condensate fields $\rho_a, \omega_a$, as well as an axion mass comparable to the condensation scale, can be generated by including a dependence of the Kähler potential $k$ (and correspondingly the function $f$) on the variables $U_a, \bar{U}_a$. Terms of the form $V^{-2n}\sum_a \left(U_a\bar{U}_a\right)^n$ and $V^{-2n}\left(U\bar{U}\right)^n$ are generated both by classical string corrections [33] and by field theory loop corrections [27]. Note that once the condensate fields are integrated out these induce, by virtue of their *vev*'s (2.14), "nonperturbative" corrections to the Kähler potential for the dilaton, of the type discussed by Banks and Dine [17]. However in the single condensate case [34] it was found that these terms are insufficient to stabilize the dilaton, and one must appeal instead to string nonperturbative effects.[3] We expect the same conclusion to hold in the multicondensate case. If this is so, the interpretation of contributions to the Kähler potential of the form $f = Ae^{-B/V}$ as arising from field theoretic corrections to our our static model may be questionable. We therefore adopt the point of view that the unknown function $f$ parameterizes string nonperturbative corrections.

In the static models studied here, cancellation of the cosmological constant by string nonperturbative corrections alone requires that they are significant at the vacuum: $< f(\ell) > \approx < 2\ell > \approx 1$. This has implications[4] for phenomenological analyses [36] of gauge coupling unification. Including nonperturbative corrections to the Kähler potential for the linear multiplet $L$, *i.e.*, taking $k(L) = \ln L + g(L)$ with $f(L)$ related to $g(L)$ as in (2.3) with $V \to L$, the two-loop boundary condition [22] on the $\overline{MS}$ gauge couplings now reads (for affine level one):

$$\begin{aligned} g_a^{-2}(\mu_s) &= g_s^{-2} + \frac{C_a}{8\pi^2}\left\{g(\ell) + \ln\left[f(\ell)+1\right] - \ln 2\right\} \\ &\quad - \frac{1}{16\pi^2}\sum_I b_a^I \ln\left[\left(t^I + \bar{t}^I\right)|\eta^2(t^I)|^2\right], \\ g_s^{-2} &= \frac{f+1}{2\ell}, \quad \mu_s^2 = \ell e^{g-1} = \frac{1}{2}e^{g-1}(f+1)g_s^2. \end{aligned} \tag{6.1}$$

Note that the tree coupling of the effective field theory is now $2g_s^{-2} =$

[3] It can be shown that the static model of [16] is indeed the low energy limit of the dynamical model of [34].

[4] Other gauge-dependent threshold corrections [35] have recently been found.

$\langle (f+1)/\ell \rangle$, and integration over the condensate fields with *vev*'s given by (2.28) gives corrections to the Kähler potential for $\ell$ of the form [17] $\sim e^{-n/b_a g_s^2} = e^{-(f+1)/b_a \ell}$, when kinetic terms for $U_a, \bar{U}_a$ are included. On the other hand, we expect string nonperturbative effects [18] to be $\sim e^{-n\pi/\sqrt{\ell}}$ since the linear supermultiplet containing the 3-form $d^{[n}b^{pq]}$ is the fundamental field in string compactifications (as opposed *e.g.* to 5-brane compactification [37], in which the dilaton is in a chiral multiplet and the moduli are in linear multiplets). If one performs a duality transformation in the usual way [14] *via* a Lagrange multiplier $S + \bar{S}$:

$$\mathcal{L} = \int \mathrm{d}^4\theta E \left[ -2 + f(L) + \frac{1}{3} (L + \Omega) \left( S + \bar{S} \right) \right],$$

where $\Omega$ is the Chern-Simons superfield, $L$ is unconstrained and $S$ is chiral, the equations of motion for $L$ give precisely $S + \bar{S} = (f+1)/L$, so that $\mathrm{Re}s$ is always the tree-level inverse squared coupling constant in the chiral formulation of the effective field theory. Including the Green-Schwarz term and loop corrections in the chiral formulation [23] again gives (6.1).

**Acknowledgements**

PB and MKG would like to acknowledge support from the Miller Institute for Basic Research in Science, and also the Aspen Center for Physics where part of this work was completed. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797.

# A. Appendix: Chiral multiplet formulation

There has been interest in the question as to whether the linear and chiral multiplet formulations are equivalent at the quantum level. They are presumably equivalent in the sense that we may perform a duality transformation at the superfield level on the Lagrangian (2.1) so as to recast it entirely in terms of chiral supermultiplets; the resulting effective Lagrangian is apt to be rather complicated. The more practical question that we address in this

appendix is the extent to which the above results can be reproduced if one takes as a starting point the usual chiral supermultiplet formalism for the dilaton with the gaugino condensates represented by unconstrained chiral supermultiplets, and naïvely generalizes the methods commonly used in this context.

In the chiral multiplet formulation, the Green-Schwarz term appears as a correction to the Kähler potential, which we take to be

$$K(S,T^I) = \ln(L) + \tilde{g}(L) + \sum_I g^I, \quad L^{-1} = S + \bar{S} - b\sum_I g^I, \tag{A.1}$$

where $\tilde{g}$ is the correction from nonperturbative string effects. Modular invariance of the Yang-Mills Lagrangian at the quantum level is assured by the transformation property of $S$ under (2.10):

$$S \to S + b\sum_I F^I, \tag{A.2}$$

and modular covariance of the Kähler potential $[K \to K + \sum_I(F^I + \bar{F}^I)]$ requires that it depend on $S$ only through the real superfield $L$. We introduce static condensate superfields $\Pi^\alpha, U_a$ as before, but now the superfield

$$U_a = e^{K/2}H_a^3 \tag{A.3}$$

does not satisfy the constraint (3.4) because $H_a$ is taken to be an unconstrained chiral superfield.[5] We construct the superpotential in analogy to (2.1), using the standard Veneziano-Yankielowicz approach:

$$W_{tot} = W_{cond} + W(\Pi), \tag{A.4}$$

where $W(\Pi)$ is the same as in (2.27), and

$$\begin{aligned} W_{cond} &= W_C + W_{VY} + W_{th}, \quad W_C = \frac{1}{4}S\sum_a H_a^3, \\ W_{VY} &= \frac{1}{4}\sum_a H_a^3\left(3b'_a \ln H_a + \sum_\alpha b_a^\alpha \ln \Pi^\alpha\right), \\ W_{th} &= \frac{1}{4}\sum_{a,I}\frac{b_a^I}{8\pi^2}H_a^3 \ln[\eta^2(T^I)], \end{aligned} \tag{A.5}$$

[5] This is probably where the departure from the approach of Section 2 is the most sensitive. The correct procedure – which is not the one usually followed – would be to use a 3-form supermultiplet description [28].

where $W_C$ represents the classical contribution. $H_a^3$ transforms in the same way as $U_a$ under rigid chiral and conformal transformations, and the anomaly matching conditions give the same constraints on the $b$'s as in Section 2. Then it is straightforward to check that under the modular transformation (2.10) with $H_a \rightarrow e^{-\sum_I F^I/3}$, we have $W_{cond} \rightarrow e^{-\sum_I F^I/3} W_{cond}$, as required for modular invariance of the Lagrangian. Summing the various contributions, the superpotential for $H_a$ can be written in the form

$$W_{cond} = \frac{1}{4} \sum_a b'_a H_a^3 \ln \left\{ e^{S/b'_a} H_a^3 \prod_\alpha (\Pi^\alpha)^{b_a^\alpha/b'_a} \prod_I [\eta(T^I)]^{-b_a^I/4\pi^2 b'_a} \right\}. \qquad \text{(A.6)}$$

The bosonic Lagrangian takes the standard form:

$$\begin{aligned} \mathcal{L}_B &= -\frac{1}{2}\mathcal{R} - \frac{1}{3} M\bar{M} + K_{i\bar{m}} \left( F^i \bar{F}^{\bar{m}} - \partial_\mu z^i \partial^\mu z^{\bar{m}} \right) \\ &\quad + e^{K/2} \left[ F^i \left( W_i + K_i W \right) - \bar{M} W + \text{h.c.} \right], \end{aligned} \qquad \text{(A.7)}$$

where $Z^i = S, T^I, H_a, \Pi^\alpha$, $z^i = Z^I|_{\theta=\bar{\theta}=0}$. In our static model $K_{i\bar{m}}, K_i = 0$ for $Z^i, Z^m = H_a, \Pi^\alpha$, and the equations of motion for $F^i$ give $W_i = 0$ for these fields. This determines the chiral superfields $H_a, \Pi^\alpha$ as holomorphic functions of $S, T^I$. Making the same restrictions on $W(\Pi)$ and the $b_a^\alpha$ as in Section 2, we obtain:

$$\begin{aligned} H_a^3 &= e^{(2n+1)i\pi(b'_a - b_a)/b_a - b'_a/b_a} e^{-S/b_a} \prod_I [\eta(T^I)]^{2(b-b_a)/b_a} \prod_\alpha |b_a^\alpha/4c_\alpha|^{-b_a^\alpha/b_a}, \\ \Pi^\alpha &= -\frac{b_a^\alpha}{4c_\alpha} H_a^3 \prod_I [\eta(T^I)]^{-2(q_I^\alpha - 1)}, \quad b_a^\alpha \neq 0. \end{aligned} \qquad \text{(A.8)}$$

As in (2.28), the correct dependence of the gaugino condensates on the gauge coupling constant $< (\text{Re}s)^{-\frac{1}{2}} >$, $s = S|_{\theta=\bar{\theta}=0}$, is recovered. Note however that in contrast to (2.28) the gaugino condensate phases are quantized once $\text{Im}s$ is fixed at its *vev*. Using these results gives

$$W_{tot} = W(S, T^I) = -\frac{1}{4} \sum_a b_a H_a^3. \qquad \text{(A.9)}$$

The effective potential is determined in the standard way after eliminating the remaining auxiliary fields through their equations of motion:

$$\begin{aligned} M &= -3e^{K/2} W, \quad \bar{F}^{\bar{m}} = -e^{K/2} K^{i\bar{m}} \left( W_i + K_i W \right), \quad Z^i = S, T^I, \\ V(s, t^I, \bar{t}^I) &= e^K \left[ K^{i\bar{m}} \left( W_i + K_i W \right) \left( \bar{W}_{\bar{m}} + K_{\bar{m}} \bar{W} \right) - 3|W|^2 \right]. \end{aligned} \qquad \text{(A.10)}$$

The inverse Kähler metric for the Kähler potential (A.1) is

$$\begin{aligned} K^{I\bar{J}} &= \frac{4(\mathrm{Re}t^I)^2}{(1-bK_s)}\delta^{IJ}, \quad K^{I\bar{s}} = -\frac{2b\mathrm{Re}t^I}{(1-bK_s)}, \\ K^{s\bar{s}} &= \frac{1-bK_s+3b^2K_{s\bar{s}}}{K_{s\bar{s}}(1-bK_s)}, \end{aligned} \tag{A.11}$$

and the potential reduces to

$$\begin{aligned} V &= \frac{e^K}{1-bK_s}\Big\{K_{s\bar{s}}^{-1}\left(1-bK_s+3b^2K_{s\bar{s}}\right)|W_s+K_sW|^2 + 4\sum_I\left(\mathrm{Re}t^I\right)^2|W_I+K_IW|^2 \\ &\quad -2b\left[\left(\bar{W}_s+K_s\bar{W}\right)\sum_I\mathrm{Re}t^I\left(W_I+K_IW\right)+\mathrm{h.c.}\right]\Big\} - 3e^K|W|^2. \end{aligned} \tag{A.12}$$

We have

$$\begin{aligned} -2\mathrm{Re}t^I\left(W_I+K_IW\right) &= -\sum_a\frac{1}{4b_a}\left[1-bK_s-\frac{b-b_a}{b_a}\mathrm{Re}t^I\zeta(t^I)\right]H_a^3, \\ W_s+K_sW &= \sum_a\frac{1}{4b_a^2}\left(1-K_sb_a\right)H_a^3, \end{aligned} \tag{A.13}$$

and the potential can be written in the form

$$V = \frac{e^K}{16(1-bK_s)}\sum_{ab}|h_ah_b|^3\cos\omega_{ab}R_{ab}, \tag{A.14}$$

where now $\omega_a$ is the phase of $h_a^3 = H_a^3|_{\theta=\bar{\theta}=0}$, $\omega_{ab}$ is defined as before, and

$$\begin{aligned} R_{ab} &= b_ab_bf_{ab}(\ell)+(b-b_a)(b-b_b)\sum_I|1+4\mathrm{Re}t^I\zeta(t^I)|^2, \quad \ell = L|_{\theta=\bar{\theta}=0}, \\ f_{ab}(\ell) &= (1-bK_s)\left[\frac{(1-b_aK_s)(1-b_aK_s)}{b_ab_bK_{s\bar{s}}}-3\right]. \end{aligned} \tag{A.15}$$

In the absence of nonperturbative effects $K_s = -\ell,\ K_{s\bar{s}} = \ell^2,\ f_{ab} \to -2b\ell$ as $\ell \to \infty$, and the potential is unstable in the strong coupling direction, as expected. A positive definite potential requires that $f_{++}(\ell)$ be postive semi-definite where, as before, $b_+$ is the largest $b_a$. Note that the perturbative expression for $f_{aa}(\ell)$ is negative for $b_a\ell > 1.4$, while in the linear multiplet

formalism, the corresponding expression is negative only for $b_a\ell > 2.4$, so nonperturbative effects are required to be more important in the chiral multiplet formulation. If there is only one condensate, the self-dual point for the moduli is again a minimum, but $< F^I > \neq 0$. In the general case, the minimization equations for the moduli read

$$\begin{aligned}\frac{\partial V}{\partial t^I} &= \frac{e^K}{16(1-bK_s)}\sum_{ab}|h_a h_b|^3\cos\omega_{ab}\left(\frac{2b}{n}\zeta(t^I)\sum_c \beta_{ca}R_{ab} + \frac{\partial}{\partial t^I}R_{ab}\right) \\ &\quad + \left(A + \frac{2b}{n}\zeta(t^I)\sum_a \frac{1}{b_a}\right)V, \end{aligned} \tag{A.16}$$

where $\beta_{ab}$ is defined as in (3.25). Again imposing $< V >= 0$, the minimum is shifted slightly away from the self dual point if some $\beta_{ab} \neq 0$.

The effective Lagrangian in the linear multiplet formalism – like the string and field theory loop-corrected Yang-Mills Lagrangian [22, 23] – depends only on the variables $t^I$ and the modular invariant field $\ell$, so the Lagrangian is invariant under modular transformations on the $t^I$ alone. In contrast, the effective Lagrangian in the standard chiral multiplet approach has an explicit $s$-dependence which accounts for the fact that the self-dual point is not the minimum. The standard chiral construction forces a holomorphic coefficient for the interpolating superfield for the Yang Mills composite superfield $U \simeq \mathrm{Tr}(\mathcal{W}^\alpha\mathcal{W}_\alpha)$, and hence cannot faithfully reflect nonholomorphic contributions from the Green-Schwarz term and field theory loop corrections. The last point can be evaded by incorporating these renormalization effects in the Kähler potential [38, 15, 32] rather than in the superpotential, in which case it is also possible to recover invariance under continuous infinitesimal S-duality rotations in the weak coupling limit. Again this a property of the Yang-Mills Lagrangian and the linear multiplet formulation of condensation, but not of the chiral multiplet formulation.[6] However, in this last approach the relation of the effective Lagrangian for condensation to the underlying Yang-Mills Lagrangian is much less transparent. We emphasize that we do not claim that there is no effective chiral Lagrangian dual to that of Section 2, with the same physics. However a straightforward approach based on the chiral multiplet formalism leads to different physics, in particular the

[6]This can again be traced [15] to the fact that the condensate superfield (A.3) does not satisfy the constraint (3.4).

nonvanishing of the moduli F-terms in the vacuum, which has implications for flavor-changing neutral currents.

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## FIGURE CAPTIONS

Fig. 1: The scalar potential $V$ (in reduced Planck units) is plotted versus $\ell$ and $\ln t$.

Fig. 2: The scalar potential $V$ (in reduced Planck units) is plotted versus $\ell$ with $t^I = 1$ (the self-dual point).

Fig. 3: The scalar potential $V$ (in reduced Planck units) is plotted versus $\ln t$ with $\ell = \langle \ell \rangle$.

# Figure 1

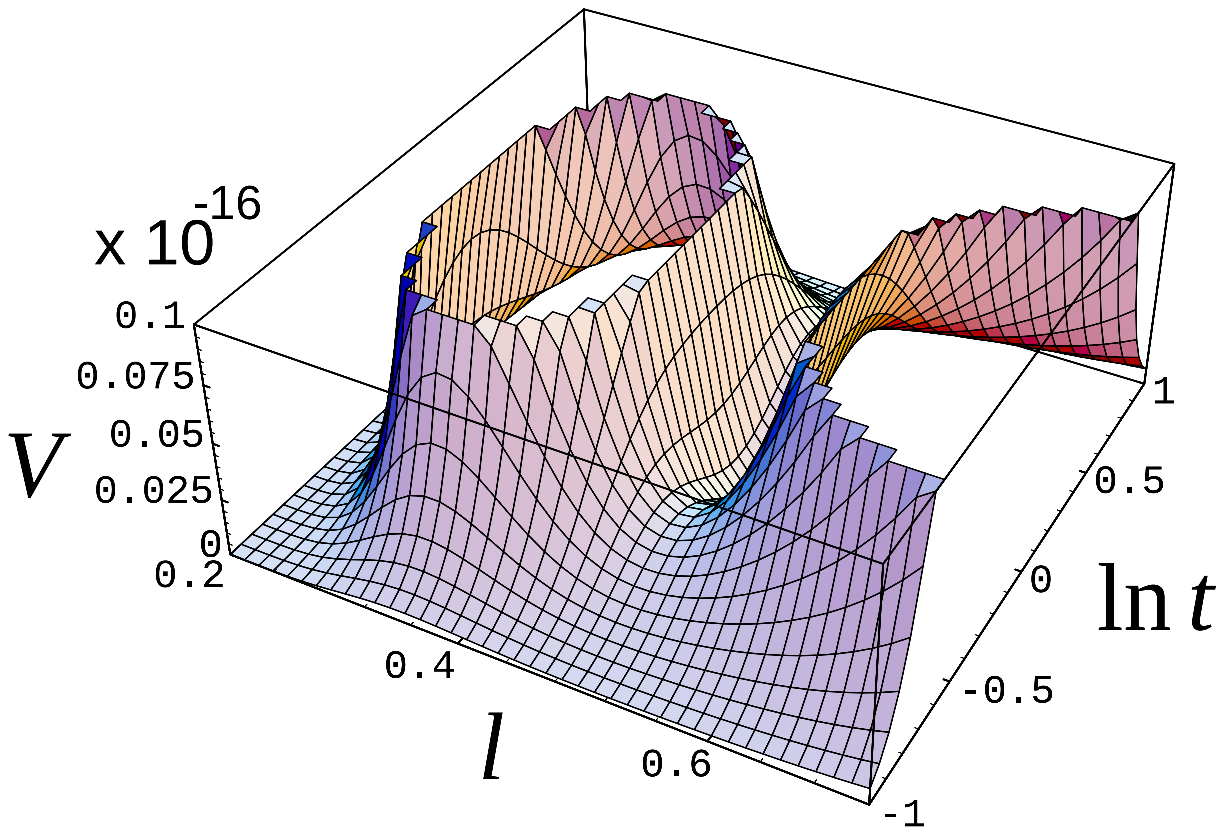

Figure 2

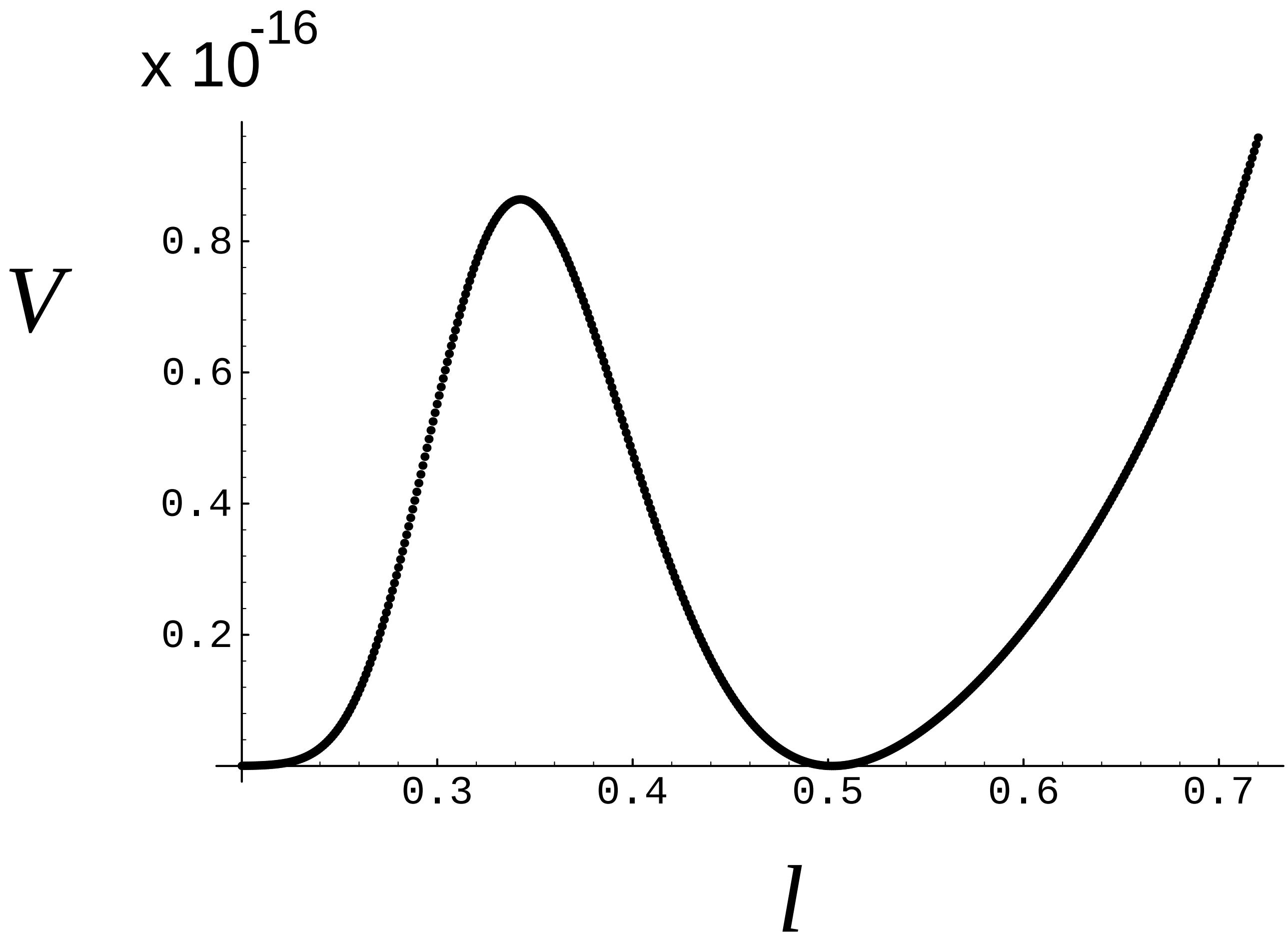

Figure 3